AQA A-Level Physics: Materials – Calculation Questions

Praneel Physics

1. Calculate the stress on a wire of cross-sectional area $2.0 \times 10^{-6}\,\text{m}^2$ under a force of $50\,\text{N}$. **(P)**

Working and Answer:

Use $\sigma = \frac{F}{A}$.
$\sigma = \frac{50}{2.0\times10^{-6}} = 2.5 \times 10^{7}\,\text{Pa}$.

2. Calculate the strain of a wire that stretches from 2.00 m to 2.02 m. **(P)**

Working and Answer: Use $\epsilon = \frac{\Delta L}{L_0}$. $\epsilon = \frac{0.02}{2.00} = 0.01$ (or 1%).

3. Calculate the force constant of a spring that extends by 0.05 m under a load of 10 N. **(P)**

Working and Answer: Use $k = \frac{F}{x}$. $k = \frac{10}{0.05} = 200\,\mathrm{N/m}$.

4. Calculate the Young's modulus for a material with stress 2×10^8 Pa and strain 0.004. **(P)**

Working and Answer:

Use $E = \frac{\sigma}{\epsilon}$.
$E = \frac{2\times10^8}{0.004} = 5 \times 10^{10}$ Pa.

5. Calculate the energy stored in a spring with $k = 500$ N/m compressed by 0.1 m. **(P)**

Working and Answer:

Use $E = \frac{1}{2}kx^2$.
$E = \frac{1}{2} \times 500 \times 0.1^2 = 2.5$ J.

6. Calculate the stress and strain for a wire ($E = 2 \times 10^{11}$ Pa) that stretches by 0.5% under a force. **(PP)**

Working and Answer:

Step 1: $\epsilon = 0.005$ (given).
Step 2: $\sigma = E\epsilon = 2 \times 10^{11} \times 0.005 = 1 \times 10^{9}$ Pa.

7. Calculate the extension and energy stored in a spring ($k = 400$ N/m) when a 20 N force is applied. **(PP)**

Working and Answer:

Step 1: $x = \frac{F}{k} = \frac{20}{400} = 0.05$ m.
Step 2: $E = \frac{1}{2}kx^2 = \frac{1}{2} \times 400 \times 0.05^2 = 0.5$ J.

8. Calculate the Young's modulus and strain for a wire ($A = 1 \times 10^{-6}\,\text{m}^2$) that extends by 0.1 mm under 100 N (original length = 2 m). **(PP)**

Working and Answer:
Step 1: $\sigma = \frac{100}{1\times10^{-6}} = 1 \times 10^{8}\,\text{Pa}$.
Step 2: $\epsilon = \frac{0.1\times10^{-3}}{2} = 5 \times 10^{-5}$. $E = \frac{\sigma}{\epsilon} = 2 \times 10^{12}\,\text{Pa}$.

9. Calculate the force and work done to compress a spring ($k = 300\,\text{N/m}$) by 0.08 m. **(PP)**

Working and Answer:
Step 1: $F = kx = 300 \times 0.08 = 24\,\text{N}$.
Step 2: $W = \frac{1}{2}kx^2 = \frac{1}{2} \times 300 \times 0.08^2 = 0.96\,\text{J}$.

10. Calculate the cross-sectional area and strain of a wire ($E = 1 \times 10^{11}$ Pa) under 500 N stress 5×10^{8} Pa. **(PP)**

Working and Answer:

Step 1: $A = \frac{F}{\sigma} = \frac{500}{5\times10^{8}} = 1 \times 10^{-6}\,\mathrm{m}^2$.
Step 2: $\epsilon = \frac{\sigma}{E} = \frac{5\times10^{8}}{1\times10^{11}} = 0.005$.

11. Calculate the stress, strain, and Young's modulus for a wire ($L_0 = 3\,\text{m}$, $A = 5 \times 10^{-7}\,\text{m}^2$) that extends by 1.5 mm under 150 N. **(PPP)**

Working and Answer:

Step 1: $\sigma = \frac{150}{5\times10^{-7}} = 3 \times 10^8\,\text{Pa}$.
Step 2: $\epsilon = \frac{1.5\times10^{-3}}{3} = 5 \times 10^{-4}$.
Step 3: $E = \frac{3\times10^8}{5\times10^{-4}} = 6 \times 10^{11}\,\text{Pa}$.

12. Calculate the spring constant, energy stored, and force required for an additional 0.02 m extension when a spring stores 0.8 J at 0.1 m extension. **(PPP)**

Working and Answer:

Step 1: $k = \frac{2E}{x^2} = \frac{1.6}{0.01} = 160\,\mathrm{N/m}$.
Step 2: $E = 0.8\,\mathrm{J}$ (given).
Step 3: $F = kx = 160 \times 0.12 = 19.2\,\mathrm{N}$.

13. Calculate the original length, strain energy, and stress for a wire ($E = 2 \times 10^{11}$ Pa, $A = 2 \times 10^{-6}\,\text{m}^2$) that stores 0.5 J when stretched by 0.2 mm. **(PPP)**

Working and Answer:

Step 1: $E_{\text{strain}} = \frac{1}{2}\sigma\epsilon V$. Find V first.
Step 2: $\epsilon = \frac{\Delta L}{L_0}$. Assume $L_0 = 1\,\text{m}$, $\epsilon = 2 \times 10^{-4}$.
Step 3: $\sigma = E\epsilon = 4 \times 10^7\,\text{Pa}$. $V = \frac{2E_{\text{strain}}}{\sigma\epsilon} = \frac{1}{4\times10^7\times2\times10^{-4}} = 0.125\,\text{m}^3$.
$L_0 = \frac{V}{A} = 62.5\,\text{m}$.

14. Calculate the force constant, work done, and strain energy per unit volume for a spring ($k = 250\,\text{N/m}$) compressed by $0.04\,\text{m}$ (volume $= 1 \times 10^{-5}\,\text{m}^3$). **(PPP)**

Working and Answer:

Step 1: $k = 250\,\text{N/m}$ (given).
Step 2: $W = \frac{1}{2}kx^2 = \frac{1}{2} \times 250 \times 0.04^2 = 0.2\,\text{J}$.
Step 3: Energy density $= \frac{W}{V} = \frac{0.2}{1\times10^{-5}} = 2 \times 10^4\,\text{J/m}^3$.

15. Calculate the stress, strain, and percentage extension for a wire ($E = 1.5 \times 10^{11}$ Pa) under 3×10^7 Pa stress. **(PPP)**

Working and Answer:

Step 1: $\sigma = 3 \times 10^7$ Pa (given).
Step 2: $\epsilon = \frac{\sigma}{E} = \frac{3\times10^7}{1.5\times10^{11}} = 2 \times 10^{-4}$.
Step 3: % extension $= \epsilon \times 100 = 0.02\%$.

16. Calculate the Young's modulus, strain energy, stress at fracture, and percentage elongation for a wire ($L_0 = 2\,\text{m}$, $A = 4 \times 10^{-7}\,\text{m}^2$) that breaks under 200 N after extending by 0.5 mm. **(PPPP)**

Working and Answer:

Step 1: $\sigma_{\text{fracture}} = \frac{200}{4\times10^{-7}} = 5 \times 10^8$ Pa.
Step 2: $\epsilon = \frac{0.5\times10^{-3}}{2} = 2.5 \times 10^{-4}$.
Step 3: $E = \frac{\sigma}{\epsilon} = 2 \times 10^{12}$ Pa.
Step 4: $E_{\text{strain}} = \frac{1}{2}\sigma\epsilon V = \frac{1}{2} \times 5 \times 10^8 \times 2.5 \times 10^{-4} \times (4 \times 10^{-7} \times 2) = 0.05\,\text{J}$.

17. Calculate the spring constant, total energy stored, force for 0.06 m compression, and elastic limit force if a spring stores 1.8 J at 0.1 m extension. **(PPPP)**

Working and Answer:

Step 1: $k = \frac{2E}{x^2} = \frac{3.6}{0.01} = 360\,\text{N/m}$.
Step 2: $E = 1.8\,\text{J}$ (given).
Step 3: $F = kx = 360 \times 0.06 = 21.6\,\text{N}$.
Step 4: Elastic limit force $= k \times 0.1 = 36\,\text{N}$.

18. Calculate the cross-sectional area, strain energy density, Young's modulus, and percentage reduction in diameter if a wire ($L_0 = 5\,\text{m}$, diameter $= 1\,\text{mm}$) extends by $0.25\,\text{mm}$ under $500\,\text{N}$ (Poisson's ratio $= 0.3$). **(PPPP)**

Working and Answer:

Step 1: $A = \pi r^2 = \pi(0.5 \times 10^{-3})^2 \approx 7.85 \times 10^{-7}\,\text{m}^2$.
Step 2: $\sigma = \frac{500}{7.85\times10^{-7}} \approx 6.37 \times 10^8\,\text{Pa}$. $\epsilon = \frac{0.25\times10^{-3}}{5} = 5 \times 10^{-5}$.
Step 3: $E = \frac{\sigma}{\epsilon} \approx 1.27 \times 10^{13}\,\text{Pa}$.
Step 4: Lateral strain $= -\nu\epsilon = -0.3 \times 5 \times 10^{-5} = -1.5 \times 10^{-5}$. % reduction $= 0.0015\%$.

19. Calculate the stress, strain, energy stored, and force for a 0.03 m extension of a spring ($k = 400\,\mathrm{N/m}$) originally 0.5 m long with cross-section $1 \times 10^{-6}\,\mathrm{m}^2$. **(PPPP)**

Working and Answer:

Step 1: $F = kx = 400 \times 0.03 = 12\,\mathrm{N}$.
Step 2: $\sigma = \frac{F}{A} = \frac{12}{1\times10^{-6}} = 1.2 \times 10^7\,\mathrm{Pa}$.
Step 3: $\epsilon = \frac{0.03}{0.5} = 0.06$.
Step 4: $E = \frac{1}{2}kx^2 = \frac{1}{2} \times 400 \times 0.03^2 = 0.18\,\mathrm{J}$.

20. Calculate the Young's modulus, strain energy, fracture stress, and new diameter under load for a wire ($E = 2 \times 10^{11}$ Pa, diameter = 0.5 mm, $\nu = 0.33$) stretched by 0.1% under 50 N. **(PPPP)**

Working and Answer:

Step 1: $\epsilon = 0.001$. $E = 2 \times 10^{11}$ Pa (given).
Step 2: $\sigma = E\epsilon = 2 \times 10^{8}$ Pa. $A = \frac{F}{\sigma} = \frac{50}{2\times10^{8}} = 2.5 \times 10^{-7}\,\text{m}^2$.
Step 3: $E_{\text{strain}} = \frac{1}{2}\sigma\epsilon V$. Assume $L_0 = 1$ m, $V = 2.5 \times 10^{-7}\,\text{m}^3$. $E = 0.025$ J.
Step 4: Lateral strain $= -\nu\epsilon = -3.3 \times 10^{-4}$. New diameter $= 0.5 \times (1 - 3.3 \times 10^{-4}) \approx 0.4998$ mm.

21. Calculate the Young's modulus, strain energy, Poisson's ratio effect, fracture force, and energy per unit volume for a wire ($L_0 = 4\,\text{m}$, diameter $= 0.8\,\text{mm}$, $\nu = 0.25$) that breaks at 0.2% strain under 500 N. **(PPPPP)**

Working and Answer:

Step 1: $\epsilon = 0.002$. $A = \pi(0.4 \times 10^{-3})^2 \approx 5.03 \times 10^{-7}\,\text{m}^2$.
Step 2: $\sigma_{\text{fracture}} = \frac{500}{5.03\times10^{-7}} \approx 9.94 \times 10^8\,\text{Pa}$.
Step 3: $E = \frac{\sigma}{\epsilon} \approx 4.97 \times 10^{11}\,\text{Pa}$.
Step 4: Lateral strain $= -0.25 \times 0.002 = -5 \times 10^{-4}$. New diameter $= 0.8 \times (1 - 5 \times 10^{-4}) = 0.7996\,\text{mm}$.
Step 5: $E_{\text{strain}} = \frac{1}{2}\sigma\epsilon V = \frac{1}{2} \times 9.94 \times 10^8 \times 0.002 \times (5.03 \times 10^{-7} \times 4) \approx 2\,\text{J}$.

22. Calculate the spring constant, total energy stored, maximum force, elastic potential energy at 0.05 m, and work done to compress further to 0.08 m if a spring stores 5 J at 0.1 m extension. **(PPPPP)**

Working and Answer:

Step 1: $k = \frac{2E}{x^2} = \frac{10}{0.01} = 1000\,\mathrm{N/m}$.
Step 2: $E = 5\,\mathrm{J}$ (given).
Step 3: $F_{\mathrm{max}} = kx = 1000 \times 0.1 = 100\,\mathrm{N}$.
Step 4: $E(0.05) = \frac{1}{2} \times 1000 \times 0.05^2 = 1.25\,\mathrm{J}$.
Step 5: $W = \frac{1}{2} \times 1000 \times (0.08^2 - 0.05^2) = 1.95\,\mathrm{J}$.

23. Calculate the stress, strain, Young's modulus, percentage reduction in cross-section, and strain energy for a wire ($L_0 = 3\,\text{m}$, diameter $= 1\,\text{mm}$, $\nu = 0.3$) stretched by $0.3\,\text{mm}$ under $200\,\text{N}$. **(PPPPP)**

Working and Answer:

Step 1: $A = \pi(0.5 \times 10^{-3})^2 \approx 7.85 \times 10^{-7}\,\text{m}^2$. $\sigma = \frac{200}{7.85\times10^{-7}} \approx 2.55 \times 10^8\,\text{Pa}$.
Step 2: $\epsilon = \frac{0.3\times10^{-3}}{3} = 1 \times 10^{-4}$. $E = \frac{\sigma}{\epsilon} \approx 2.55 \times 10^{12}\,\text{Pa}$.
Step 3: Lateral strain $= -0.3 \times 1 \times 10^{-4} = -3 \times 10^{-5}$. % reduction $= 0.003\%$.
Step 4: $E_{\text{strain}} = \frac{1}{2}\sigma\epsilon V = \frac{1}{2} \times 2.55 \times 10^8 \times 1 \times 10^{-4} \times (7.85 \times 10^{-7} \times 3) \approx 0.03\,\text{J}$.

24. Calculate the force constant, energy stored at 0.04 m extension, additional work to extend to 0.07 m, maximum safe force, and strain energy density for a spring ($k = 600\,\text{N/m}$, elastic limit $= 0.1\,\text{m}$). **(PPPPP)**

Working and Answer:

Step 1: $k = 600\,\text{N/m}$ (given).
Step 2: $E(0.04) = \frac{1}{2} \times 600 \times 0.04^2 = 0.48\,\text{J}$.
Step 3: $W = \frac{1}{2} \times 600 \times (0.07^2 - 0.04^2) = 0.99\,\text{J}$.
Step 4: $F_{\text{safe}} = 600 \times 0.1 = 60\,\text{N}$.
Step 5: Assume volume $V = 1 \times 10^{-5}\,\text{m}^3$, energy density $= \frac{0.48}{V} = 4.8 \times 10^4\,\text{J/m}^3$.

25. Calculate the Young's modulus, strain energy, Poisson's ratio effect, fracture stress, and new length under load for a wire ($E = 1.8 \times 10^{11}$ Pa, diameter = 0.6 mm, $\nu = 0.35$, $L_0 = 2$ m) stretched by 0.15 mm under 100 N. **(PPPPP)**

Working and Answer:

Step 1: $\epsilon = \frac{0.15\times10^{-3}}{2} = 7.5 \times 10^{-5}$. $E = 1.8 \times 10^{11}$ Pa (given).
Step 2: $\sigma = E\epsilon = 1.35 \times 10^7$ Pa. $A = \frac{100}{1.35\times10^7} \approx 7.41 \times 10^{-6}\,\text{m}^2$.
Step 3: Lateral strain $= -0.35 \times 7.5 \times 10^{-5} = -2.625 \times 10^{-5}$. New diameter $= 0.6 \times (1 - 2.625 \times 10^{-5}) \approx 0.59998$ mm.
Step 4:
$E_{\text{strain}} = \frac{1}{2}\sigma\epsilon V = \frac{1}{2} \times 1.35 \times 10^7 \times 7.5 \times 10^{-5} \times (7.41 \times 10^{-6} \times 2) \approx 7.5 \times 10^{-3}$ J.
Step 5: New length $= 2 + 0.15 \times 10^{-3} = 2.00015$ m.